***Valorile funcțiilor trigonometrice în primul cadran :***

| x | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ |
|---|---|---|---|---|---|
| sinx | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cosx | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| tgx | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | / |
| ctgx | / | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |

***Semnele functiilor trigonometrice şi monotonia pe cadrane:***

| *x* | *I* | *II* | *III* | *IV* |
|---|---|---|---|---|
| *sinx* | + | + | – | – |
| *cosx* | + | – | – | + |
| *tgx* | + | – | + | – |
| *ctgx* | + | – | + | – |

| *x* | *I* | *II* | *III* | *IV* |
|---|---|---|---|---|
| *sinx* | ↗ | ↘ | ↘ | ↗ |
| *cosx* | ↘ | ↘ | ↗ | ↗ |
| *tgx* | ↗ | ↗ | ↗ | ↗ |
| *ctgx* | ↘ | ↘ | ↘ | ↘ |

***Identități fundamentale***

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$tg\left(\frac{\pi}{2} - x\right) = ctgx \qquad ctg\left(\frac{\pi}{2} - x\right) = tgx$$

$$\sin^2 x + \cos^2 x = 1 \qquad tgx \cdot ctgx = 1$$

***Reducerea la primul cadran***

| $II \to I$ | $III \to I$ | $IV \to I$ |
|---|---|---|
| $\sin x = \sin(\pi - x)$, | $\sin x = -\sin(x - \pi)$, | $\sin x = -\sin(2\pi - x)$ |
| $\cos x = -\cos(\pi - x)$, | $\cos x = -\cos(x - \pi)$, | $\cos x = \cos(2\pi - x)$ |
| $tgx = -tg(\pi - x)$, | $tgx = tg(x - \pi)$, | $tgx = -tg(2\pi - x)$ |
| $ctgx = -ctg(\pi - x)$, | $ctgx = ctg(x - \pi)$, | $ctgx = -ctg(2\pi - x)$ |

***Formule paritate***

$$\sin(-x) = -\sin x \qquad \arcsin(-x) = -arcsinx$$
$$\cos(-x) = \cos x \qquad \arccos(-x) = \pi - \arccos x$$
$$tg(-x) = -tgx \qquad arctg(-x) = -arctgx$$
$$ctg(-x) = -ctgx \qquad arcctg(-x) = \pi - arcctgx$$

***Formule periodicitate***

$$\sin(2k\pi + x) = \sin x \qquad tg(k\pi + x) = tgx$$
$$\cos(2k\pi + x) = \cos x \qquad ctg(k\pi + x) = ctgx \quad , k \in \mathbb{Z}$$

***Formule pentru sume şi diferențe de unghiuri***

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$
$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$tg(x+y) = \frac{tgx + tgy}{1 - tgx \cdot tgy}$$
$$tg(x-y) = \frac{tgx - tgy}{1 + tgx \cdot tgy}$$
$$ctg(x+y) = \frac{ctgx \cdot ctgy - 1}{ctgy + ctgx}$$
$$ctg(x-y) = \frac{ctgx \cdot ctgy + 1}{ctgy - ctgx}$$

***Formule pentru unghiuri duble***

$$\sin 2x = 2\sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$tg 2x = \frac{2tgx}{1 - tg^2 x} \qquad ctg 2x = \frac{ctg^2 x - 1}{2ctgx}$$

***Formule pentru unghiuri triple***

$$\sin 3x = 3\sin x - 4\sin^3 x \qquad \cos 3x = 4\cos^3 x - 3\cos x$$
$$tg3x = \frac{3tgx - tg^3 x}{1 - 3tg^2 x} \qquad ctg3x = \frac{ctg^3 x - 3ctgx}{3ctgx - 1}$$

***Formule pentru jumătăți de unghiuri***

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \qquad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$
$$tg\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x} \qquad ctg\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{1-\cos x}} = \frac{\sin x}{1-\cos} = \frac{1+\cos x}{\sin x}$$

***Formule pentru substituția cu*** $t = tg\frac{x}{2}$

$$\sin x = \frac{2tg\,\frac{x}{2}}{1+tg^2\,\frac{x}{2}} = \frac{2t}{1+t^2} \qquad \cos x = \frac{1\text{-}tg^2\,\frac{x}{2}}{1+tg^2\,\frac{x}{2}} = \frac{1-t^2}{1+t^2} \qquad \text{unde } t = tg\,\frac{x}{2}$$

$$\text{tg}x = \frac{2\text{tg}\,\frac{x}{2}}{1\text{-tg}^2\,\frac{x}{2}} = \frac{2t}{1-t^2} \qquad \text{ctg}x = \frac{1\text{-tg}^2\,\frac{x}{2}}{2\text{tg}\,\frac{x}{2}} = \frac{1-t^2}{2t}$$

***Formule pentru trasformarea sumelor în produse***

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \qquad \sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} \qquad \cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin x + \cos x = \sin x + \sin\left(\frac{\pi}{2} - x\right) = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$$

***Formule pentru trasformarea produselor în sume***

$$\sin x \cdot \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\cos x \cdot \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$$\sin x \cdot \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

**Probleme**

**1)** Arătați că:

**a)** $\dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$

**b)** $2\sin^2 x - 1 = 1 - 2\cos^2 x$

**c)** $(\cos x - \sin x)^2 + (\cos x + \sin x)^2 = 2$

**2)** Arătați că:

**a)** $\sin 2x = \dfrac{2tgx}{1+tg^2 x}$

**b)** $\cos 2x = \dfrac{1-tg^2 x}{1+tg^2 x}$

**c)** $tg\,2x = \dfrac{2tgx}{1-tg^2 x}$

**3)** Arătați că:

**a)** $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

**b)** $\sin(x+y)-\sin(x-y)=2\sin y\cos x$

**c)** $\sin(x+y)+\cos(x-y)=(\sin x+\cos x)(\sin y+\cos y)$

**4)** Arătați că:

**a)** $\cos(x+y)+\cos(x-y)=2\cos x\cos y$

**b)** $\cos(x+y)-\cos(x-y)=-2\sin x\sin y$

**c)** $\cos(x+y)-\sin(x-y)=(\cos x-\sin x)(\sin y+\cos y)$

**5)** Arătați că:

**a)** $\sin(x+y)\cdot\sin(x-y)=\sin^2 x-\sin^2 y$

**b)** $\cos(x+y)\cdot\cos(x-y)=\cos^2 x-\sin^2 y$

**c)** $\sin\left(x+\frac{\pi}{2}\right)\cdot\cos\left(x-\frac{\pi}{2}\right)=\sin x\cos x$

**6)** Arătați că:

**a)** $1+\sin 2x=(\sin x+\cos x)^2$

**b)** $1-\sin 2x=(\sin x-\cos x)^2$

**c)** $(\sin x+\cos x)(\sin x-\cos x)=-\cos 2x$

**7)** Arătați că:

**a)** $\frac{1+\cos 2x}{2}=\cos^2 x$

**b)** $\frac{1-\cos 2x}{2}=\sin^2 x$

**c)** $1-2\sin^2 x=2\cos^2 x-1$

**8)** Arătați că:

**a)** $\sin^4 x+\cos^2 x=\sin^2 x+\cos^4 x$

**b)** $\sin^3 x+\cos^3 x=(\sin x+\cos x)(1-\sin x\cos x)$

**c)** $\sin^6 x+\cos^6 x=1-\frac{3}{4}\sin^2 2x$

**9)** Arătați că:

**a)** $\cos x\cos 2x=\frac{\sin 4x}{4\sin x}$

**b)** $\cos x\cos 2x\cos 4x=\frac{\sin 8x}{8\sin x}$

**c)** $\cos x\cos 2x\cos 4x\cos 8x=\frac{\sin 16x}{16\sin x}$

**10)** Arătați că:

**a)** $(1+\cos x)(1-2\cos x)^2=\cos 3x+1$

**b)** $(\sin x+\sin 2x)(2\cos x-1)=\sin 3x$

**c)** $(\cos x+\cos 2x)(2\cos x-1)=\cos 3x+1$

**11)** Arătați că următoarele expresii sunt constante:

**a)** $E=\cos^2 x\left(\sin^2 x+tg^2 x+\cos^2 x\right)$

**b)** $E=\sin^4 x+\cos 2x-\cos^4 x$

**c)** $E=\sqrt{\sin^4 x+4\cos^2 x}+\sqrt{\cos^4 x+4\sin^2 x}$

**12)** Calculați :

**a)** $E=\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

**b)** $E=\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$

**c)** $E=\cos\frac{\pi}{31}\cos\frac{2\pi}{31}\cos\frac{4\pi}{31}\cos\frac{8\pi}{31}\cos\frac{16\pi}{31}$

**13)** Calculați :

**a)** $E=\frac{\sin 9^o}{\sin 3^o}-\frac{\cos 9^o}{\cos 3^o}$

**b)** $E=\frac{\sin 18^0}{\sin 6^o}-\frac{\cos 18^0}{\cos 6^o}$

**c)** $E=\frac{\sin 36^o}{\sin 12^o}-\frac{\cos 36^o}{\cos 12^o}$

**14)** Calculați :

**a)** $E=\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}$

**b)** $E=\cos^2\frac{\pi}{12}+\cos^2\frac{3\pi}{12}+\cos^2\frac{5\pi}{12}+\cos^2\frac{7\pi}{12}+\cos^2\frac{9\pi}{12}+\cos^2\frac{11\pi}{12}$

**c)** $E=\sin^2 1^o+\sin^2 2^o+...+\sin^2 90^o$

**15)** Calculați :

**a)** $E=\cos^2 20^o\cdot\cos^2 40^o\cdot\cos^2 60^o\cdot\cos^2 80^o$

**b)** $E=\sin^2 10^o\cdot\sin^2 30^o\cdot\sin^2 50^o\cdot\sin^2 70^o$

**c)** $E=\sin^2 1^o\cdot\sin^2 3^o\cdot...\cdot\sin^2 89^o$